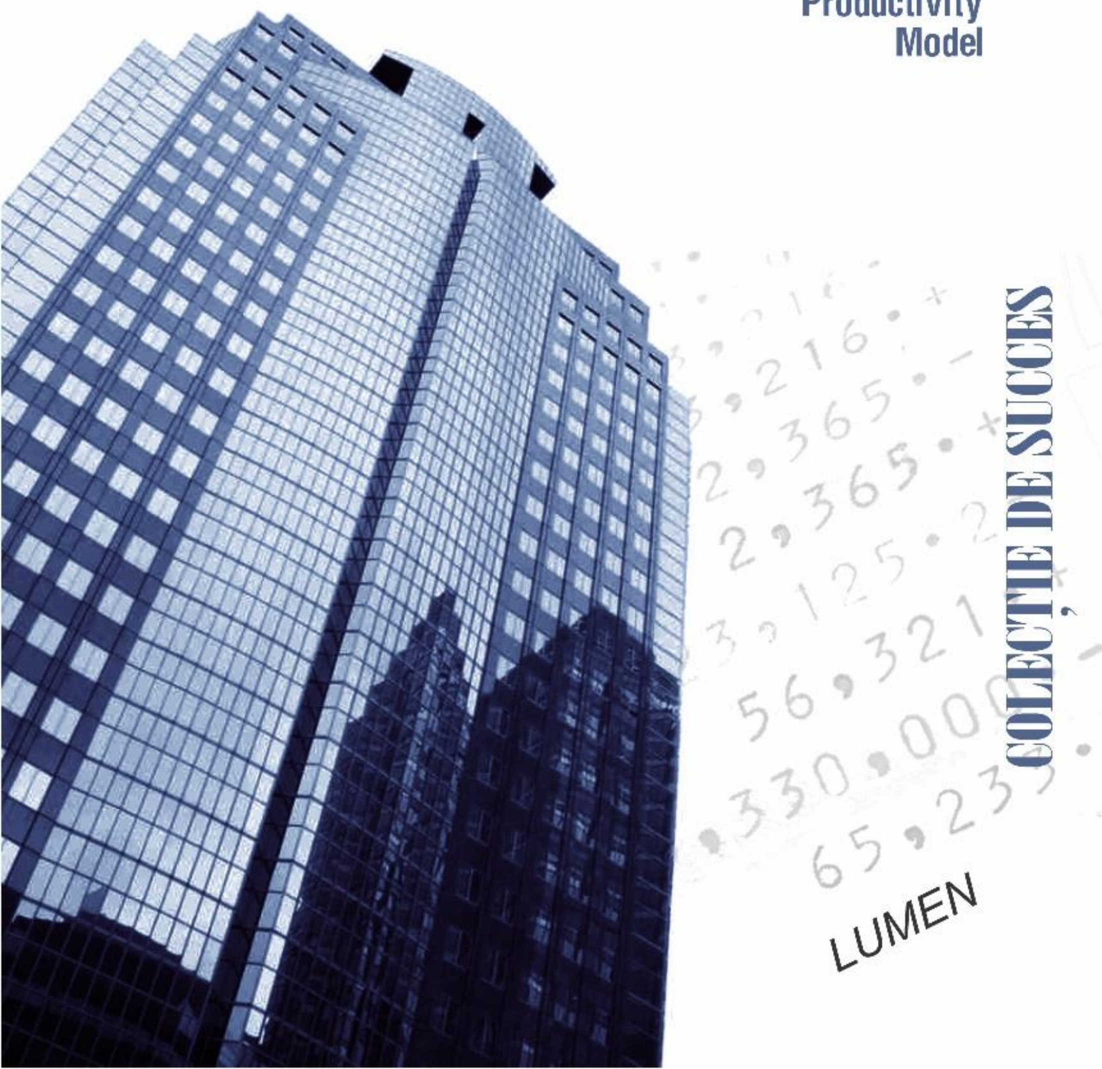
Sebastian Buhai

A Comparative Perspective

Wages, Seniority and Separation Rates in a Stochastic Productivity Model



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Wages, Seniority and Separation Rates in a Stochastic Productivity Model: A Comparative Perspective

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Abstract

We discuss a theoretical framework for the job duration of individual workers and the evolution of the wage rate during that job assuming that individual productivity follows a geometric Brownian. A comparative overview of job search, random learning and random growth models is put forward as background to the literature on job tenure distribution. The random growth model fits best the hump-shaped tenure profile observed in data on job separation rates and is consistent with empirical evidence that log wages follow a random walk. We provide a synopsis of the persisting debate on the returns to job seniority adding a non-deterministic tenure profile perspective. The specification of the model allows the application of option theory to calculate the value of a job and the optimal job separation rule. An extension to the initial model adding log firm size is introduced.

1 Introduction

This paper targets the ever recurring themes of the job tenure distribution, wage rate evolution and the relationship between job seniority and individual wages. Though a lot of work has been undertaken in this field, the mission has not been accomplished yet: the factors triggering job separation remain still unclear to our day while the debate on whether we do or we do not have tenure profiles on wages seems to continue relentlessly. We undertake a relatively new approach assuming a model with an unpredictable evolution of the productivity match after the start of a job, rooted in the random growth productivity framework developed in Teulings and Van der Ende (2000). Starting a job demands some specific investments; when the match productivity does not evolve favorably, the investments lose their value and separation becomes the only efficient alternative.

Most theories on the determinants of the distribution of job tenures have focused on search and learning models. In search models workers keep their present job until they find a better one. The chances of finding an even better job diminish as the selection process proceeds. Hence in the search framework job duration increases with the worker's labour market experience, e.g. Jovanovic (1979b), Burdett and Mortensen (1998). In random learning models a worker and a firm start a job without knowing the quality of their match, this quality being revealed in time. The productivity has a constant match-specific mean. Workers periodically observe productivity and quit to another job whenever their mean productivity is below a standard that increases with tenure. This process continues until the workers find jobs in which their inferred productivity will be above-standard, e.g. Jovanovic (1979a), Miller (1984).

Far less attention has been paid to models with a random evolution of job productivity. Such a model assumes a stochastic evolution of match productivity after the date of the job start. In particular, individual productivity is assumed to follow a geometric Brownian motion (a continuous-time random walk in logarithms). Specific investments such as hiring costs or firm-specific training costs are required at job start. When the match productivity falls below a threshold, the worker and the firm separate. The model has strong predictive accuracy in underlying empirical data. Teulings and Van der Ende (2000) have shown in this sense that the random growth productivity model can explain data observations on tenure distribution better than search or random learning models. Furthermore, by immediate implication of the model, the logarithm of the firm size and the logarithm of the wages would also follow a random walk. Both these findings are supported empirically: first, the firm size evolves approximately according to Gibrat's law: Jovanovic (1982) provides well-documented evidence that the Gibrat law tends to hold for large firms; second, several papers strongly support the fact that log wages approximately follow a random walk, e.g. Abowd and Card (1989) or Topel and Ward (1992). Given all the evidence on tenure distributions, evolution of firm size and wages, we find surprising that the random walk assumption has not been applied before in this context. A model of random proportional growth at firm-level employment scale has been derived by Bentolila and Bertola (1990). The close relationship between this model and the random productivity model at individual level has been revealed in Teulings and van der Ende (2000). In Bentolila and Bertola's model a firm is increasingly uncertain about the productivity of its employees in a more distant future and any random shocks to future productivity are in fact shocks in the firm's demand curve. We will revisit in this paper Teulings and van der Ende's model next to submitting an extension of the original set-up to include firm size. The generalized model can account in an elegant way for implications of Kuhn's (1989) last-in-first-out (LIFO) layoff rule and it can provide a start for further research on insider-outsider theories of the labour market.

Teulings and Van der Ende used the random growth model to explore *inter alia* the relationship between job seniority and wages. There is by now a considerably large literature on job tenure profiles in wages: e.g. Altonji and Shakotko (1987), Abraham and Farber (1987), Topel (1991), Altonji and Williams (1997), Farber (1999).

The conclusions are however diverging. Some authors have suggested that tenure profiles might be fully explained by selection bias, since good jobs and good workers survive. The consequence would then be that the high wage of high tenure workers is merely an artefact, not being due to job seniority but rather to favorable characteristics of the respective workers. Other authors claim that wage-tenure profiles are empirically important and that the wage increases due to additional years of tenure are large. We shall therefore include in this paper a synopsis of the debate on the returns to job seniority. Given the high interest on the subject in existing literature, it is striking that no one has explicitly considered the possibility of a non-deterministic tenure profile, as implied by the random growth productivity model. The survival of a selective sample of random walks generates a tenure profile that is partially consistent with Topel (1991) and that at the same time puts under fire much of the previous work.

The methodology of the model discussed in this paper is based on the application of option theory. We mentioned already that specific investments when the job starts are required; this leads to irreversible hiring and separation decisions. By analogy one can attach option values to these investments. Given the geometric Brownian motion used as underlying productivity path, we are able to apply Dixit's (1989) option theory to calculate the value of a job and to derive optimal job separation rules in a similar way as within the theory of financial options. In particular, the use of option theory provides a powerful theoretical apparatus for the analysis of the firm's optimal strategy, similar to Bentolila and Bertola's (1990) framework. The model can be enabled, as an extension, to test and treat implications of hold up problems and insurance issues, given that Dixit's theory can be immediately applied in a non-risk-neutral environment.

We will structure the paper as follows: a background discussion on several theories of tenure distribution and job exit rates will be the subject of the 2^{nd} section; section 3. will present empirical evidence on wage rate progress and will overview a much controversial debate over the wage returns to seniority; in section 4. we will highlight the specifications of the random growth model, extend these initial specifications to include firm size as well and discuss comparatively links with existing literature; finally a summary of this study and further research avenues are put forward in the last section.

2 Models of tenure distribution and optimal separation

2.1 Search models

The bulk of studies on labour market dynamics made extensive use of search or learning models. On a closer scrutiny the search framework seems to have actually been the long-time favoured one. In a typical search environment a worker faces an individual labor market, as reviewed in Lippman and McCall (1976). In his labour market a worker without relevant additional outside options, for instance a male worker in the working age range, may at any moment be offered a job. Ignoring non-pecuniary gains, the duration of a job is typically related to the job offer arrival rate and a wage distribution. Therefore in search models we have two types of stochastic shocks that might influence the separation decision. The first shock is the arrival process of new employment offers and the second is, conditional on the arrival, the value of those offers. We have separation when the value of a recent offer exceeds the value of the current job. Both shocks are modeled as transitory shocks in the job search literature. This means that the probabilities of job offer arrivals are not correlated over time. A major implication of the job-search theory is that the number of offers received by a worker increases with the time he spends on the labor market. If however the best of all these offers is his current job, then in expectation the maximum will increase with experience. Hence the probability of receiving an even better job and thus of separating declines with experience; this generally underlines the empirical findings in the relative long-run, such as for instance quarterly or annual basis. Nonetheless empirical research

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shows that the probability of leaving a job is actually increasing in the first several months of employment and decreasing thereafter. Or standard search models do not account for this initial increase in exit rates with tenure. A more detailed structural model is required to explain this phenomenon; in particular learning and random growth models have been proved to better fit the empirical data, especially on the short term.

We consider one of the standard search models in the literature, Jovanovic (1979b), and discuss its main assumptions. The model by Jovanovic (1979b) is a model of permanent separations, which the author includes under the category of "pure-search-goods" models of job change. Similar types of models have been previously discussed in Burdett (1978) or Mortensen (1978). Next to on-the-job search intensity Jovanovic considers firm-specific human capital investment, his paper focusing on the relationship between firm-specific human capital and the likelihood of future job separations. The worker's search intensity determines the arrival rate of new wage offers. These new offers are drawn independently from the wage-offer distribution characterized by the cumulative distribution function $F(w)$. We denote by $\lambda(t)\Delta t + o(\Delta t)$ the probability that a wage offer will arrive during the time interval $(t, t + \Delta t)$. Conditional on the distribution $F(w)$, the worker's optimal policy is characterized by a reservation wage $\theta(t)$. The job ends therefore as soon as a wage offer exceeding $\theta(t)$ is received. If we further define the survivor function as

$$\overline{F}(w) \equiv 1 - F(w) \tag{1}$$

and

$$h(t) \equiv \lambda(t)\overline{F}[\theta(t)] \tag{2}$$

then we will have $h(t)\Delta t + o(\Delta t)$ as the probability that an acceptable offer arrives on the interval $(t, t + \Delta t)$. Assume that the fraction $s(t)$ of the worker's time is devoted to on-the-job search while another fraction, $\phi(t)$, is devoted to on-the-job training, with $s(t) + \phi(t) \in [0, 1]$. Denote by $x(t)$ the worker's productivity on a particular job, where one can write

$$x(t) = \mu + k(t) \tag{3}$$

μ is the quality of the employer-worker match possessing a distribution $F(\mu)$ across matches and $k(t)$ is the human capital stock accumulated through training on the job. Jovanovic makes the crucial assumption that the productivity $x(t)$ evolves according to the following law

$$\frac{dx(t)}{dt} = g[\phi(t)x(t)] - \delta x(t),\ x(0) = \mu \tag{4}$$

where $g(0) = 0$, $g'(.) > 0$, and $g''(.) < 0$. This equation states that at $t = 0$, the productivity of the worker is equal to μ; afterwards productivity can be increased by doing on-the-job training. If no time is devoted to this investment (if $\phi(t) = 0$), productivity depreciates at a rate δ. The worker's wage is assumed to be equal to his net marginal product, where the actual amount produced by the worker is proportional to the fraction of time $(1 - \phi - s)$ that he decides to spend working. Hence,

$$w(t) = [1 - \phi(t) - s(t)]x(t) \tag{5}$$

As one can notice, Jovanovic does assume that all training and search costs are to be paid by the worker; the worker also gets all the rents associated with being well matched and those associated with a particular human-capital stock and this while other previous models yield the conclusions that it would be optimal for the rents associated with a good match to be shared between the worker and the employer, e.g. Mortensen (1978). This apparently unique rent sharing assumption is nevertheless not essential in the light of Jovanovic's (1979b) main rationale; he argues that even if the assumption that all the rents go to the worker were totally unacceptable, the results of his paper would still be relevant since they do in fact characterize that particular turnover, job-search, and respectively human capital-investment behavior that will maximize each worker's lifetime discounted expected marginal product[1].

Jovanovic's model also postulates that the accumulated on-the-job training is completely firm specific, general human capital being ignored here for the sake of simplicity. The separation condition is inferred from equations (3) and (4) above:

$$\mu' > \mu + k(t) \tag{6}$$

where μ' is the quality of the match with a potential new employer, while μ and $k(t)$ are the current match quality and respectively current job accumulated human-capital stock. The change in the productivity evolution equation (4) is that the initial condition becomes $x(0) = \mu'$. Jovanovic further defines the following: $V[x(t), t]$ as the value to the worker of having a productivity equal to $x(t)$ at t, with $0 \leq t \leq T$; $R(t)$ as the probability that the current job episode will end before calendar time t. Having $h(t)$ defined in (2), we can write

$$R(t) = 1 - e^{-\int_{t_0}^{t} h(y)dy} \tag{7}$$

The wage offer arrival rate is by hypothesis increasing and concave in the fraction of time spent searching, $s(t)$: $\lambda = \lambda[(s(t)]$, $\lambda(0) = 0$, $\lambda' > 0$, $\lambda'' < 0$. By setting marginal cost of search equal to marginal cost of return and performing a few derivations, Jovanovic obtains the following:

$$x(\tau) = \lambda'[s(\tau)] \int_{x(\tau)}^{\infty} \{V(y, \tau) - V[x(\tau), \tau]\} f(y)dy \tag{8}$$

In order to interpret expression (8) one can differentiate totally with respect to $x(\tau)$ while holding τ constant:

[1] Presumably there are many different sharing arrangements that lead to exactly the behaviour in this paper; Jovanovic mentions Mortensen (1978) as having addressed this issue.

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$$D \equiv \left.\frac{ds\,(\tau)}{dx(\tau)}\right|_{\tau \text{ constant}} = \frac{\lambda'[s(\tau)]}{x(\tau)\lambda''[s(\tau)]}\left\{1 + \lambda'[s(\tau)]V_x[x(\tau),\tau]\overline{F}[x(\tau)]\right\} \tag{9}$$

Since we had $\lambda' > 0$ and $\lambda'' < 0$ by hypothesis, we obtain from (9) that

$$D \equiv \left.\frac{ds\,(t)}{dx(t)}\right|_{t \text{ const}} < 0 \tag{10}$$

It appears thus that the amount of time devoted to search for alternative employment, $s(t)$, decreases with $x(t)$ holding t fixed. Jovanovic's conclusion is therefore that those who are better matched and those that have more specific human capital spend less time searching. He also verifies that separation probabilities as a function of tenure are uniformly lower for those who are well matched for two reasons: firstly, from (10), those workers who are well matched spend less time searching for alternative work, and secondly, when they do receive alternative offers, they are less likely to accept them. Certainly one main problem with this conclusion and with the model assumptions is that "being well matched" or "being badly matched" is fixed; in other words the employer-worker match value is exogenously set, both parties knowing it with certainty since the moment they start their employment relationship. As formalized in (4), individual productivity can only change with the amount of the job training undertaken (depreciating at a given rate if there is no investment in this sense) or if the worker quits to another job characterized by a better "match parameter". Or this setting is too rigid to describe a dynamic labour market where there is uncertainty about the future productivity. The problem is in fact common to conventional job search models.

We have thus seen that although Jovanovic's (1979b) is a " classical" in terms of job search models, it does gain mathematical elegance and ease in interpretation at the expense of a rigid assumptions set. We will shortly characterize a more recent search model, different in

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certain aspects from the model in Jovanovic, the wage posting game in Burdett and Mortensen (1998). Let us consider a continuum of homogeneous employers that choose permanent wage offers and a continuum of homogeneous workers[2] that search by randomly and sequentially sampling from this set of offers. The measure of workers is m,while the measure of employers is normalized to 1. Burdett and Mortensen model the unemployment alternative, having that at any moment in time each worker is either in state 0 (unemployed) or in state 1 (employed). At random time intervals a worker receives information about a new or alternative job opening. The rate of arrival is characterized by a Poisson process and it depends on the worker's current state; λ_i is the parameter of the Poisson arrival process with $i \in \{0, 1\}$. An offer is assumed to be the realization of a random draw from F, the distribution of wages among employers. As in Jovanovic, workers must respond to offers as they arrive and there is no recall. Workers move from lower to higher paying jobs when opportunity arises (jobs are identical apart from the associated wage) but they also move from employment to unemployment and viceversa. A particular assumption of the model is made for the rate of separation: job-worker matches are destroyed at an exogenous positive rate δ. Furthermore, any unemployed worker receives flow benefits b per instant. All agents discount future earnings at rate r. Given this framework, the expected discounted lifetime income of an unemployed worker, V_0, can be expressed as the solution to the following asset pricing equation:

$$rV_0 = b + \lambda_0 \left[\int \max\{V_0, V_1(\widetilde{x})\} dF(\widetilde{x}) - V_0 \right] \tag{11}$$

Equation (11) states that the opportunity cost of searching while unemployed is equal to income while unemployed plus the expected capital gain attributable to searching for an acceptable job, where

[2] The worker heterogeneity case is also tackled in Burdett and Mortensen (1998) but we limit ourselves to their initial model where all workers and all firms are respectively identical. For our purpose relaxing the homogeneity assumption is not essential.

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acceptance occurs only if the value of employment, $V_1(x)$, exceeds that of continued search. In a similar way one can obtain that the expected lifetime income of a worker currently employed at wage rate w is the current income plus the expected gain from searching for a better job minus the loss in the eventuality of getting unemployed:

$$rV_1(w) = w + \lambda_1 \int [\max\{V_1(w), V_1(\widetilde{x})\} - V_1(w)] dF(\widetilde{x}) + \delta[V_0 - V_1(w)] \tag{12}$$

A reservation wage R is further introduced such that

$$V_1(w) \gtreqless V_0 \text{ as } w \gtreqless R \text{ where } V_1(R) = V_0 \tag{13}$$

Using the results in expressions (11), (12) and (13), Burdett and Mortensen (1998) are able to derive

$$R - b = [\lambda_0 - \lambda_1] \int_R^\infty \left[\frac{1 - F(x)}{r + \delta + \lambda_1(1 - F(x))} \right] dx \tag{14}$$

Letting the ratio of the discount factor to the arrival rates of jobs for the unemployed pool tending to zero, $r/\lambda_0 \to 0$, the expression above can be simplified to

$$R - b = [k_0 - k_1] \int_R^\infty \left[\frac{1 - F(x)}{1 + k_1[1 - F(x)]} \right] dx \tag{15}$$

where $k_0 = \lambda_0/\delta$ and $k_1 = \lambda_1/\delta$ represent the ratios of state-dependent arrival rates to the job separation rate.

Given a reservation wage R, the flow of workers in and out of unemployment is straightforward. In the steady state, the flow into employment equals the flow from employment to unemployment,

$$\lambda_0[1 - F(R)]u = \delta(m - u) \tag{16}$$

Using equation (16) one can solve for the equilibrium unemployment rate:

$$u = \frac{m}{1 + \frac{\lambda_0}{\delta}[1 - F(R)]} \tag{17}$$

Burdett and Mortensen (1998) are further using the number of employed workers receiving a wage no greater than w at time t, i.e. $G(w,t)(m-u(t))$, where $G(w,t)$ is the proportion of employed workers at t receiving a wage no greater than w and $u(t)$ is the measure of unemployed at t, and compute its time derivative. This is written as the difference between the unemployed workers' flow into the labour market for wages no greater than w and the flow into unemployment or respectively into higher paying jobs:

$$\frac{dG(w,t)(m-u(t))}{dt} = \lambda_0 \max\{F(w)-F(R),0\}u(t) \\ -[\delta+\lambda_1(1-F(w))]G(w,t)(m-u(t)) \quad (18)$$

Then for all $w \geqslant R$,using (17) and the above, one can write down the unique steady-state distribution of wages earned by employed workers:

$$G(w) = \frac{F(w)-F(R)}{[1+\frac{\lambda_1}{\delta}(1-F(w))][1-F(R)]} \quad (19)$$

Focusing on the steady-state, the number of workers earning a wage in the interval $[w-\varepsilon, w]$ is represented by $[G(w)-G(w-\varepsilon)](1-u)$, while $F(w)-F(w-\varepsilon)$ is the measure of firms offering a wage in the same interval, where ε is an arbitrarily small positive quantity. Then the measure of workers per firm earning a wage w is given by:

$$l(w|R,F) = \lim_{\varepsilon\to 0} \frac{G(w)-G(w-\varepsilon)}{F(w)-F(w-\varepsilon)}(m-u) \quad (20)$$

Further assuming that $l(w|R,F) = 0$ if $w < R$ and writing $F(w) = F(w^-)+\nu(w)$ where $\nu(w)$ is the mass of firms offering wage w, we get:

$$l(w|R,F) = \frac{mk_0\frac{1+k_1(1-F(R))}{1+k_0(1-F(R))}}{[1+k_1(1-F(w))][1+k_1(1-F(w^-))]}, \text{ for } w \geqslant R \quad (21)$$

Ioan Sebastian Buhai (n. 1979, Cluj-Napoca) este admis cu bursă completă de studii la University College Utrecht, Olanda, imediat dupa absolvirea Liceului de Informatică "Tiberiu Popoviciu" din Cluj. Termină studiile universitare în 2001 cu două licențe obținute simultan: una în ştiințe exacte, cu specializarea în fizică teoretică şi matematică, şi una în ştiințe sociale, cu focus pe economie, drept internațional şi ştiințe politice. Este admis imediat în programul de studii postuniversitare în economie/econometrie al Institutului Tinbergen şi Universității Erasmus din Rotterdam, Olanda, unde începe doctoratul în economie în septembrie 2002, obținând un master de cercetare în primăvara lui 2003. Din octombrie 2005 este şi cercetător asociat la Departamentul de Economie şi Centrul pentru Performanță Corporativă al Aarhus School of Business, Danemarca. În primăvara lui 2005 este invitat la Departamentul de Economie al University College London şi Institutul pentru Studii Fiscale din Londra, Anglia, colaborând în proiecte de cercetare în microeconometrie. Interesele sale principale în cercetarea economică/econometrică sunt economia pieței muncii, econometria aplicată şi economia rețelelor. A prezentat lucrări ştiințifice la importante conferințe din domeniul economic, inter alia în cadrul congreselor anuale ale Econometric Society, European Economic Association sau Society of Labor Economists, precum şi în seminarii în universități şi instituții de cercetare de prim rang din Europa şi Statele Unite. Se pregăteşte să publice în curând lucrările recent finisate în jurnale ştiințifice de mare impact în economie/econometrie. A rămas în acelaşi timp pasionat de matematică, informatică şi drept internațional/european, discipline în care şi-a dovedit abilitatea prin câştigarea a importante premii şi distincții în olimpiade şi concursuri preuniversitare sau studențeşti, la nivel național şi internațional- şi în care continuă să exerseze şi în prezent când timpul îi permite. Face parte din cei 33 de tineri cercetători de top selectați de ziarul Cotidianul în cadrul campaniei "Generația Aşteptată". Pe lângă interesele academice, Buhai are la activ un număr însemnat de proiecte şi inițiative în politica şi managementul ştiinței, în jurnalism, iar recent este unul dintre coordonatorii unei enciclopedii online a culturii române moderne. Deşi se consideră cosmopolit, locuind în Olanda, Anglia sau Danemarca, vizitând o bună parte din restul lumii şi fiind fluent în câteva limbi de circulație internațională, Clujul va rămâne întotdeauna "acasă" pentru Sebastian Buhai. Se întoarce cu bucurie la Cluj de câte ori are ocazia şi este decis să colaboreze cu toți cei ambițioşi în transformarea citadelei transilvănene într-o metropolă europeană. Pentru mai multe detalii: www.sebastianbuhai.com

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